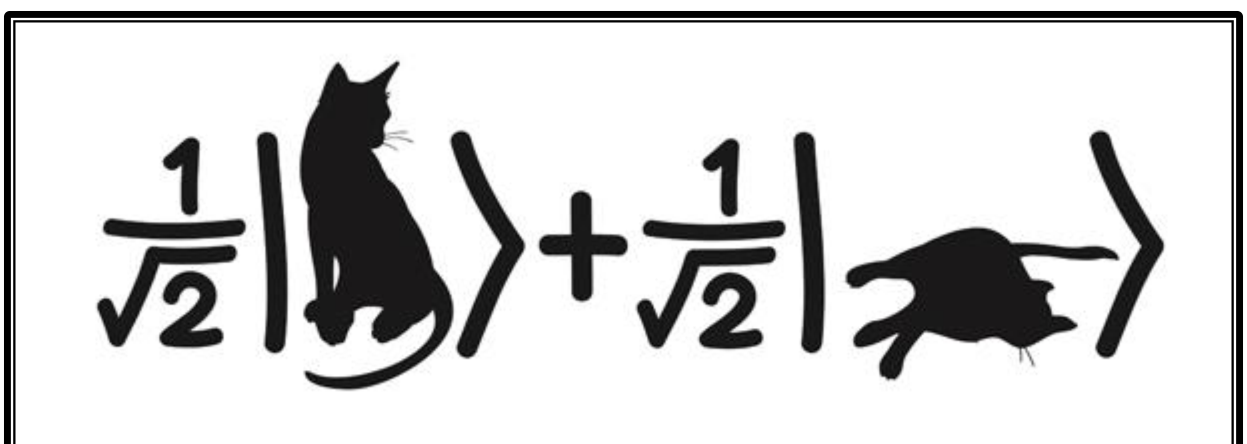
Macroscopic Quantum Systems

Quantum Puzzles
and
Double-well Potentials



Anthony James Leggett, 26 March 1938 (age 78) Camberwell, London, England

Sir Anthony J. Leggett, Professor of Physics, has been a faculty member at Illinois since 1983. He is widely recognized as a world leader in the theory of low-temperature physics, and his pioneering work on superfluidity was recognized by the 2003 Nobel Prize in Physics. He is a member of the National Academy of Sciences, the American Philosophical Society, the American Academy of Arts and Sciences, the Russian Academy of Sciences (foreign member), and is a Fellow of the Royal Society (U.K.), the American Physical Society, and the American Institute of Physics. He is an Honorary Fellow of the Institute of Physics (U.K.). He was knighted (KBE) by Queen Elizabeth II in 2004 “for services to physics”.

Macroscopic Quantum Systems and the Quantum Theory of Measurement

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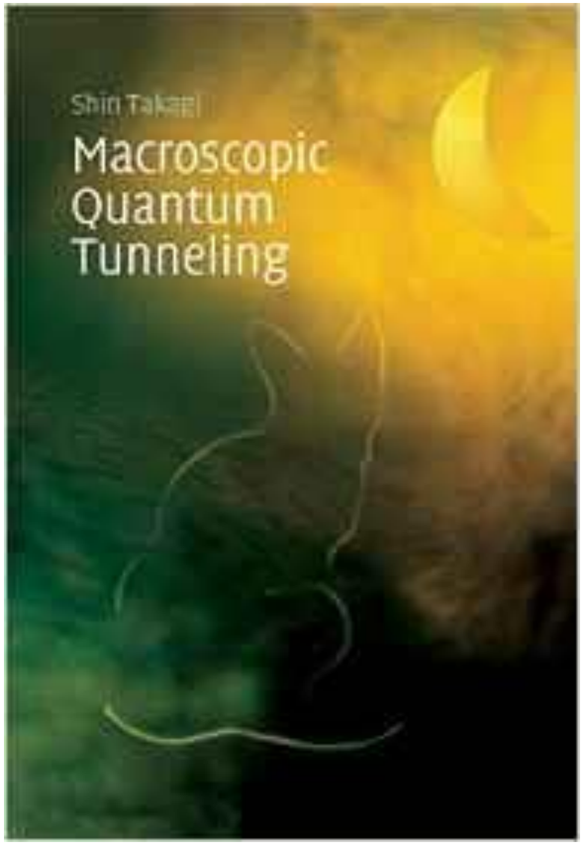
This paper discusses the question: How far do experiments on the so-called "macroscopic quantum systems" such as superfluids and superconductors test the hypothesis that the linear Schrödinger equation may be extrapolated to arbitrarily complex systems? It is shown that the familiar "macroscopic quantum phenomena" such as flux quantization and the Josephson effect are irrelevant in this context, because they correspond to states having a very small value of a certain critical property (christened "disconnectivity") while the states important for a discussion of the quantum theory of measurement have a very high value of this property. Various possibilities for verifying experimentally the existence of such states are discussed, with the conclusion that the most promising is probably the observation of quantum tunnelling between states with macroscopically different properties. It is shown that because of their very high "quantum purity" and consequent very low dissipation at low temperatures, superconducting systems (in particular SQUID rings) offer good prospects for such an observation.

§ 1. Introduction

It is a great pleasure to dedicate this paper to Professor Ryogo Kubo on the occasion of his sixtieth birthday, and to wish him many more happy



Macroscopic Quantum Tunneling,
Cambridge University Press

Shin Takagi, Fuji Tokoha University,
August 2002

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Position-Momentum Uncertainty Relation for an Open Macroscopic Quantum System

Hamid Reza Naeij & Afshin Shafiee

The macroscopic quantum systems are considered as a bridge between quantum and classical systems. In this study, we explore the validity of the original Heisenberg position–momentum uncertainty relation for a macroscopic harmonic oscillator interacting with environmental micro-particles. Our results show that, in the quasi-classical situation, the original uncertainty relation does not hold, when the number of particles in the environment is small. Nonetheless, increasing the environmental degrees of freedom removes the violation bounds in the regions of our investigation.

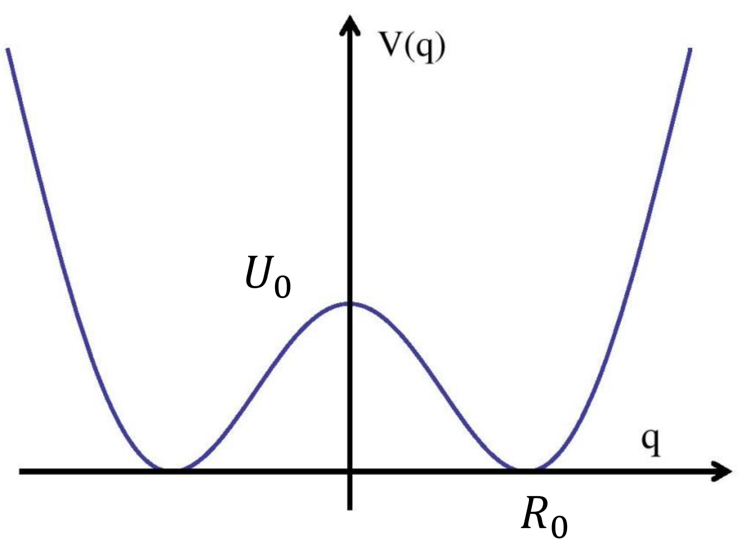
$$\tau_0 = \frac{R_0}{(U_0/M)^{\frac{1}{2}}}$$

$$q = \frac{R}{R_0}, \qquad p = \frac{P}{P_0}, \qquad t = \frac{T}{\tau_0}$$

$$V(\hat{q}) = \frac{U(\hat{R})}{U_0}, \qquad \hat{H}_s = \frac{\hat{H}_S}{U_0}$$

$$i\hbar \frac{d\psi(t)}{dt} = \hat{H}_s \psi(t)$$

$$\hat{H}_s = \frac{\hat{p}^2}{2} + V(\hat{q})$$



$$\bar{h} = \frac{\hbar}{U_0 \tau_0} = \frac{\hbar}{P_0 R_0} = \left\{ \frac{\hbar^2}{M U_0 R_0^2} \right\}^{\frac{1}{2}}$$

$$[\hat{q}, \hat{p}] = i\bar{h}$$

$$\bar{h} = \lambda_0 / R_0$$

$$H = \frac{p^2}{2} + \frac{1}{2}\omega^2 q^2 + \sum_\alpha \left[\frac{p_\alpha^2}{2} + \frac{\omega_\alpha^2}{2} x_\alpha^2 - \omega_\alpha^2 \gamma_\alpha q x_\alpha \right]$$

$$\psi_0(x_{+\alpha}, x_{-\alpha}) = \prod_{\alpha=1}^{N} \left(\frac{\omega_{+\alpha}\omega_{-\alpha}}{\pi^2 \bar{h}^2} \right)^{\frac{1}{4}} \exp\left(\frac{-\omega_{+\alpha} x_{+\alpha}^2}{2\bar{h}} \right) \exp\left(\frac{-\omega_{-\alpha} x_{-\alpha}^2}{2\bar{h}} \right)$$

$$P(q) = \frac{1}{\pi \bar{h}} \left[\frac{\omega_{+1}\omega_{-1}\omega_{+2}\omega_{-2}}{(a^2\omega_{+1} + b^2\omega_{-1})(a^2\omega_{+2} + b^2\omega_{-2})} \right]^{\frac{1}{2}}$$
$$\times \exp\left[\frac{-q^2\left[(a^2\omega_{+1}\omega_{+2})(\omega_{-1} + \omega_{-2}) + (b^2\omega_{-1}\omega_{-2})(\omega_{+1} + \omega_{+2})\right]}{\bar{h}(a^2\omega_{+1} + b^2\omega_{-1})(a^2\omega_{+2} + b^2\omega_{-2})} \right]$$

$$\bar{h} = \lambda_0 / R_0 < \lambda_\alpha / R_0$$

$$0.01 \leq \bar{h} = {\lambda_0}/{R_0} < 0.1$$

$$\Delta q^2 \Delta p^2 \geq \frac{\bar{h}^2}{4}$$

Range of violation for *N=2*:

$$0.04 < \bar{h} < 0.1$$

$$\Delta q^2 = \frac{1}{2^{5/2}}\left(\frac{\hbar}{\pi\omega_\alpha}\right)^{1/2} \qquad \Delta p^2 = \frac{1}{2^{5/2}}\left(\frac{\hbar\omega_\alpha}{\pi}\right)^{1/2}$$

$$\Delta q^2 \Delta p^2 = \frac{\hbar}{2^5\pi} < \frac{\hbar^2}{4}$$

$$\frac{1}{N^3\pi^{N-1}} < \hbar^{N-1}$$

Langevin Equation for a Dissipative Macroscopic Quantum System: Bohmian Theory versus Quantum Mechanics

Hamid Reza Naeij & Afshin Shafiee

In this study, we solve analytically the Schrodinger equation for a macroscopic quantum oscillator as a central system coupled to a large number of environmental micro-oscillating particles. Then, the Langevin equation is obtained for the system using two approaches: Quantum Mechanics and Bohmian Theory. Our results show that the predictions of the two theories are inherently different in real conditions. Nevertheless, the Langevin equation obtained by Bohmian approach could be reduced to the quantum one, when the vibrational frequency of the central system is high enough compared to the maximum frequency of the environmental particles.

$$H = \frac{p^2}{2} + \frac{1}{2}\omega^2 q^2 + \sum_\alpha \left[\frac{p_\alpha^2}{2} + \frac{\omega_\alpha^2}{2} x_\alpha^2 - \omega_\alpha^2 \gamma_\alpha q x_\alpha \right]$$

$$\psi_0(q, x_1, ..., x_N) = \prod_{\alpha=1}^{N} \left(\frac{\omega_{+\alpha}\omega_{-\alpha}}{\pi^2 \bar{h}^2} \right)^{\frac{1}{4}} \exp\left(\frac{-\omega_{+\alpha} x_{+\alpha}^2}{2\bar{h}} \right) \exp\left(\frac{-\omega_{-\alpha} x_{-\alpha}^2}{2\bar{h}} \right)$$

Quantum Approach

$$\frac{d^2 q(t)}{dt^2} = -\tilde{V}'(q(t)) + \sum_{\alpha=1}^{N} \omega_\alpha^2 \left[f'_\alpha(q(t)) x_\alpha \right]$$

$$\tilde{V}(q(t)) = V(q) + \frac{1}{2} \sum_{\alpha=1}^{N} \omega_\alpha^2 \left(f_\alpha(q) \right)^2$$

$$\ddot{q}(t) + V'\big(q(t)\big) + M(t) = \langle R(t) \rangle_{env}$$

$$M(t) = \int_{t_0}^{t} T(t - t')\dot{q}(t')dt'$$

$$T(t) = \frac{2}{\pi} \int_0^\infty \frac{d\omega_e}{\omega_e} J(\omega_e) cos\omega_e t \quad J(\omega_e) \simeq \lambda'_Q \omega_e e^{-\omega_e/\omega_c}$$

$$\ddot{q}(t) + 2\omega_0 \lambda_Q \dot{q}(t) + V'\big(q(t)\big) \simeq 0$$

$$\ddot{q}_Q(t) + 2\omega_0 \lambda_Q \dot{q}_Q(t) + \omega_0^2 q_Q(t) \simeq 0$$

Bohmian Approach

$$\psi(q, x_1, ..., x_N, t) = R\exp(iS/\bar{h})$$

$$\frac{\partial S}{\partial t} + \frac{1}{2}(\frac{\partial S}{\partial q})^2 - \frac{\bar{h}^2}{2}\frac{1}{R}\frac{\partial^2 R}{\partial q^2} + \sum_{\alpha=1}^{N}\frac{(\overrightarrow{\nabla}_{\alpha} S)^2}{2} - \frac{\bar{h}^2}{2}\sum_{\alpha=1}^{N}\frac{\hat{\nabla}_{\alpha}^2 R}{R} + V = 0$$

$$\frac{\partial R^2}{\partial t} + \frac{\partial}{\partial q}(R^2\frac{\partial S}{\partial q}) + \sum_{\alpha=1}^{N}\overrightarrow{\nabla}_{\alpha}\cdot(R^2\overrightarrow{\nabla}_{\alpha} S) = 0$$

$$Q(q, x_1, ..., x_N, t) = -\frac{\bar{h}^2}{2}\frac{1}{R}\frac{\partial^2 R}{\partial q^2} - \frac{\bar{h}^2}{2}\sum_{\alpha=1}^{N}\frac{\hat{\nabla}_{\alpha}^2 R}{R}$$

$$\ddot{q} = -\frac{\partial}{\partial q}(Q + V)$$

$$V\left(q(t), x_1(t), ..., x_N(t)\right) = \frac{1}{2}\omega^2 q^2 + \sum_{\alpha=1}^{N}\left[\frac{\omega_\alpha^2}{2}x_\alpha^2 - \gamma_\alpha q \omega_\alpha^2 x_\alpha\right]$$

$$\ddot{q}_B + 2\xi\lambda_B\dot{q}_B + \xi^2 q_B = \eta\left(x_1(t), ..., x_N(t)\right)$$

$$\ddot{q}_B(t) + 2\xi\lambda_B\dot{q}_B(t) + \xi^2 q_B(t) = 0$$

$$q_Q(t) = Ce^{-\omega_0 \lambda_Q t} sin(\sqrt{1-\lambda_Q^2}\omega_0 t + \phi)$$

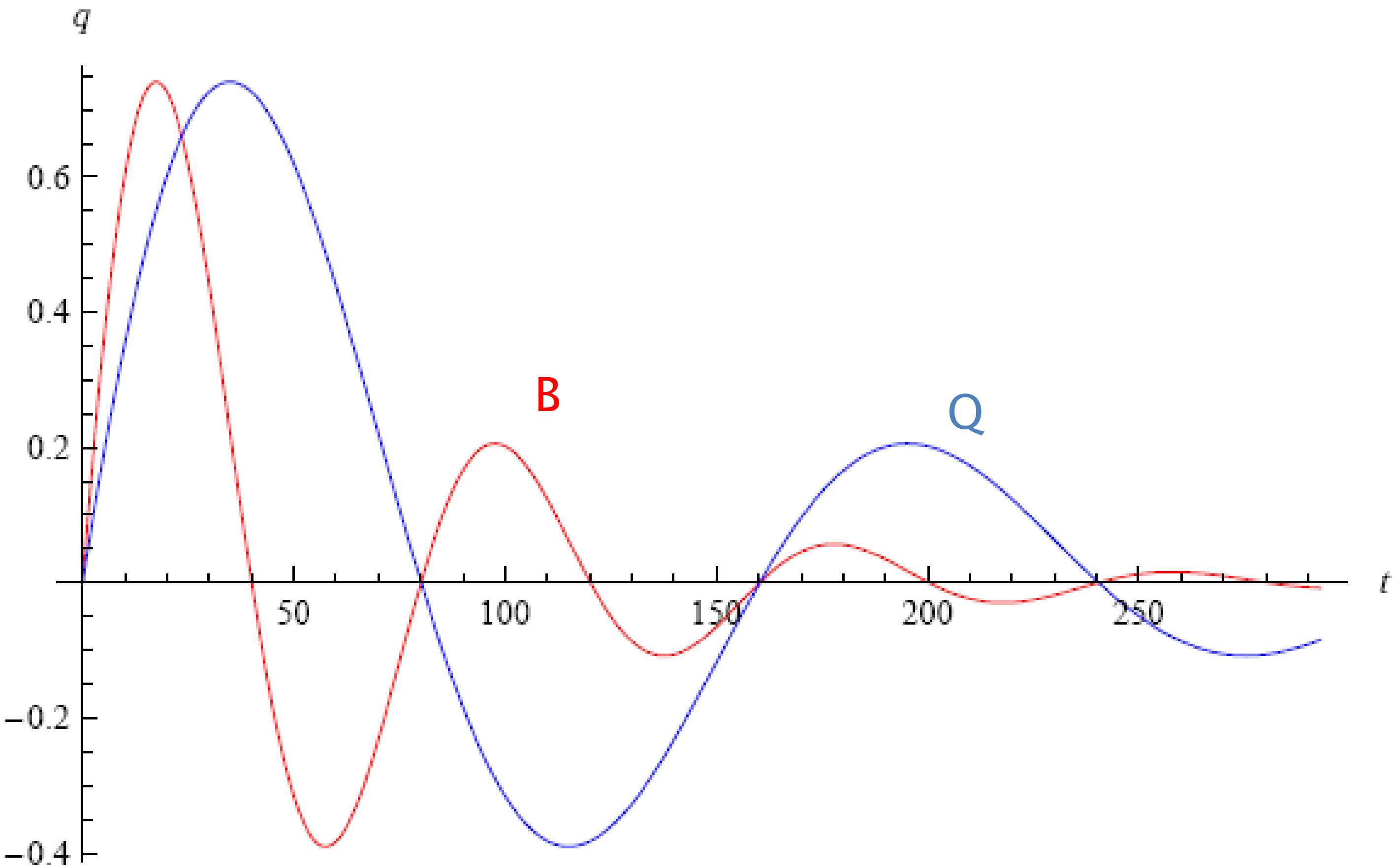
$$q_B(t) = C' e^{-\xi \lambda_B t} sin(\sqrt{1-\lambda_B^2}\xi t + \phi')$$

$$\xi^2 = -A^2 + \omega^2 - a^2 b^2 \sum_{\alpha=1}^{N} \frac{1}{N^2}(\omega_{+\alpha} - \omega_{-\alpha})^2$$

$$\omega = [\omega_0^2 + \sum_\alpha \omega_\alpha^2 \gamma_\alpha^2]^{\frac{1}{2}}$$

Similar Predictions:

$$\xi^2 \approx \omega_0^2. \qquad q(t) = e^{-\omega_0 \lambda t}[sin(\sqrt{1-\lambda^2}\omega_0 t)(\frac{\dot{q}(0)}{\sqrt{1-\lambda^2}\omega_0})]$$



For MQSs, $\omega_0 < \omega_C$ and $\xi^2 \approx \dfrac{2}{\pi}\lambda'_Q \omega_C$ with $\lambda'_Q = 2\omega_0 \lambda_Q$, we choose

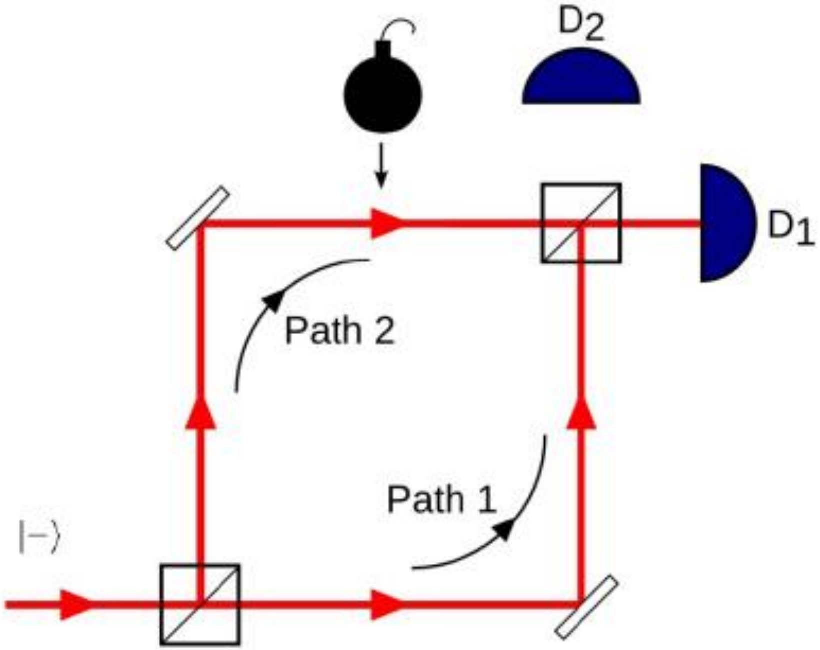
$\lambda_Q \approx 0.2,\ \omega_0 \approx 4\times 10^{10}\, s^{-1}$ (=0.04) and $\omega_C \approx 10^{12}\, s^{-1}$ (=1).

Interaction-Free Measurement For A Macroscopic Quantum System Under Decoherence

Nasim Shahmansoori & Afshin Shafiee

We consider a macroscopic quantum system in a Mach-Zehnder setup to describe an interaction-free measurement. Before reaching two detectors, the system is allowed to interact with a collection of environmental micro-oscillators. Dissipative effects caused by the decoherence process change the probabilities of the free measurement. For a macroscopic two-level system, at an appropriate time domain, there is a considerable chance of detection of a macro-object without interacting with it, due to the leakage of information.

Elitzur, A.C. & Vaidman, L., *Found .Phys.* (1993) **23**, 987. doi:10.1007/BF00736012



Elitzur–Vaidman setup

Beam Splitter Operation:

$$|-\rangle \longrightarrow \frac{1}{\sqrt{2}}(|-\rangle + i|+\rangle)$$

$$|+\rangle \longrightarrow \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$$

Mirror Operation:

$$|-\rangle \longrightarrow i|+\rangle$$

$$|+\rangle \longrightarrow i|-\rangle$$

$$|-\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}\left(|-\rangle + i|+\rangle\right) \xrightarrow{Mirrors} \frac{1}{\sqrt{2}}\left(i|+\rangle - |-\rangle\right) \xrightarrow{BS_2} \frac{1}{2}\left(i|+\rangle - |-\rangle - |-\rangle - i|+\rangle\right) = -|-\rangle$$

$$|-\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}\left(|-\rangle + i|+\rangle\right) \xrightarrow{Mirrors/Bomb} \frac{1}{\sqrt{2}}\left(i|+\rangle + i|absorb\rangle\right) \xrightarrow{BS_2} \frac{i}{2}|+\rangle - \frac{1}{2}|-\rangle + \frac{i}{\sqrt{2}}|absorb\rangle$$

Including the effect of the environment at last step:

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$
$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$\hat{H}_s|0\rangle = -\bar{h}\Omega/2 \text{ and } \hat{H}_s|1\rangle = +\bar{h}\Omega/2.$$

$$|i\rangle = |-\rangle \xrightarrow{BS_1} |\alpha\rangle \xrightarrow{Mirror/Bomb} |\beta\rangle \xrightarrow{BS_2} |\alpha'\rangle \xrightarrow{TimeEvolution} |f\rangle$$

With bomb: $P_+(t) = P_-(t) = 1/4$

Without bomb:

$$P_-^0(t) = \frac{1}{2}(1 + e^{-\Gamma_1 t/2}\cos\Omega t)$$
$$P_+^0(t) = \frac{1}{2}(1 - e^{-\Gamma_1 t/2}\cos\Omega t)$$

$$P_+^0(t) = P(D_2 \& IL \& \sim FM)$$

$$P_+(t) \ = \ P(D_2 \& IL \& FM) + P(D_2 \& IL \& \sim FM)$$

$$P(D_2 \& IL \& FM) = \frac{1}{4} - P_+^0$$

We need $P_+^0 \to 0$. Then, for all 25% events where D_2 clicks, we will be sure that the bomb has been present and free measurement is occurred.

$$Q = e^{-\Gamma_1 t/2} \cos \Omega t, \ \Gamma_1 t / 2 \sim \eta(\Omega t) / \bar{h} \sim 0.01, \eta \approx \bar{h} \approx 0.1,$$

$$\Omega \approx 10^{12} s^{-1}, \ t \approx 10^{-15} s \text{ (10 femtoseconds)}$$


thank
you